MODERNIZATION OF TURBULENCE MODELS FOR AXISYMMETRIC JET FLOW

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This article examines two models of turbulence that are in wide use: the one-parameter model described in [1] and the two-parameter model described in [2]. The modifications to these models proposed here will make it possible to appreciably more accurately describe the mixing of an axisymmetric jet.

1. Interpretation of the Prandtl Theory for Plane and Axisymmetric Jet Flows. The system of steady-state Reynolds equations for an isobaric turbulent jet flow of an incompressible liquid (density ρ = const) has the form

$$\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}=\frac{1}{y^i}\frac{\partial}{\partial y}\left(-y^i\overline{u'v'}\right),\quad \frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}=0,$$

where i = 0, 1 in the plane and axisymmetric cases; the superimposed bar denotes averaging. We close the system by means of the Boussinesq hypothesis

$$-\overline{u'v'}=\varepsilon\partial\bar{u}/\partial y$$

(ε is the eddy viscosity coefficient). To construct a theory of turbulent transport for ε, Prandtl [3] used conditional reasoning in regard to the transport of large-scale particles of fluid (conditionally referred to as moles) by a random pressure fluid. This theory has since been called the "mixing length" theory. Following Prandtl, we will examine the mechanism of turbulent transport for ε by using the example of the turbulent flow of fluid with uniform shear ($\bar{w}=0,\ \bar{v}=0,\ \partial\bar{u}/\partial x=0,\ \partial\bar{u}/\partial y=\text{const}\neq 0$). Let a certain mole obtain momentum in the transverse direction y. As a result of this, its transverse component of velocity becomes equal to v', while the mole itself is displaced on the characteristic "mixing length" ℓ. In undergoing this displacement, the mole displaces the mole previously located in its new position. We will assume that the longtitudinal component of the velocity of the mole remains unchanged during its displacement. The difference between the longitudinal components of the velocity of the displacing and displaced moles will be $u' \approx -l\,\partial\bar{u}/\partial y$. Now we can easily obtain an estimate for the correlation $\overline{u'v'}\approx-\overline{lv'}(\partial\bar{u}/\partial y)$ of interest to us. Thus, in accordance with the "mixing length" theory, the eddy viscosity coefficient is the correlation $\overline{\ell v'}$. The same result is obtained in the axisymmetric case. In the present study, we obtained a difference between the plane and axisymmetric cases thanks to the following modification of the mixing-length theory.

Let us examine the motion of a mole in a plane yOz perpendicular to the vector of mean velocity. Following the logical reasoning behind the mixing length theory, we introduce the characteristic distance ℓ_1 over which the mole is displaced due to the random pulsative action of the pressure fluid on it. We assume that the mole has the form of a sphere with the radius $R = \ell_1/2$ and can be displaced in any direction in the plane yOz. One of the possible final positions of the mole is shown in Fig. 1 by the dashed line.

It should be noted that the above assumption (ℓ_1 = 2R) means that the Lagrangian and Eulerian spatial scales of turbulence are equal. However, experimental data [4] indicates that there is a substantial difference between these scales. For jet flow, the ratio of the Lagrangian scale to the Eulerian scale is roughly 0.6.

The region within which the mole can move is represented by a circle with the radius 3R (Fig. 1). The site where the given mole might end up can initially be occupied by a mole whose center has the ordinate (y_0 + 2R) or (y_0 − 2R). At first, these moles belong to the regions represented by horizontal bands I and II in Fig. 1. We will henceforth be

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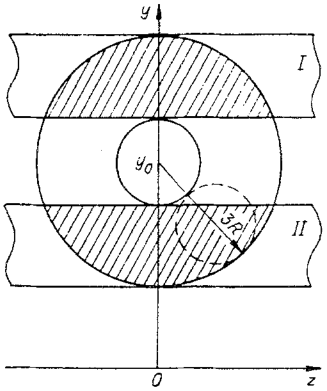

Fig. 1

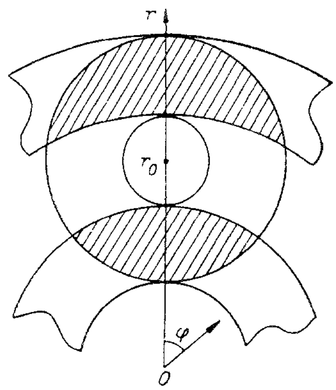

Fig. 2

interested in the region of intersection of these bands and the circle of radius 3R (hatched in Fig. 1). In the plane case, the area of each hatched region $S_0 = R^2[9\arctan(\sqrt{8}) - \sqrt{8}] \approx 8.25R^2$.

A different pattern will be seen in the axisymmetric turbulent flow of fluid with uniform shear, when

$$\bar{w} \equiv 0,\ \bar{v} \equiv 0,\ \partial\bar{u}/\partial x \equiv 0,\ \partial\bar{u}/\partial r \equiv \text{const} \neq 0.$$

Here, $r \equiv \sqrt{y^2+z^2}$. We will examine the pulsative motion of a mole in the plane $r0\varphi$ (where φ is the azimuthal angle). Regions which are horizontal bands in the plane case will become bent bands having the same width 2R as previously. The boundaries of the bent bands are circles (Fig. 2). The regions of intersection of the bent bands and the circle of radius 3R, hatched in Fig. 2, will now be unequal. The area S_1 of the upper region will be greater than the area of the lower region S_2. We establish the following connection between these areas and the approximate probabilities σ_1 and σ_2 for the displacements of the given mole and positive and negative values of v', respectively:

$$\sigma_1 = S_1/(S_1+S_2),\ \sigma_2 = S_2/(S_1+S_2).$$

We perform an approximate averaging of the pulsations of the longitudinal component of velocity, using the probabilities σ_1 and σ_2 as weight factors:

$$\bar{u'} \approx \sigma_1\left(-l_1\frac{\partial\bar{u}}{\partial r}\right) + \sigma_2\left(l_1\frac{\partial\bar{u}}{\partial r}\right) = l_1\frac{\partial\bar{u}}{\partial r}\frac{S_2-S_1}{S_2+S_1}. \tag{1.1}$$

Whereas such averaging yields a zero value for $\bar{u}'$ in the plane case, in the axisymmetric case we obtain a nontrivial profile $u_e(r) \equiv \bar{u}'(r)$. Strictly speaking, this result is incompatible with the notion of pulsation.

We introduce the effective profile of the longitudinal component of velocity u_m by means of the relation $u_m + u_e = \bar{u}$. Having made use of (1.1), after performing the necessary transformations we obtain

$$\left|\frac{\partial u_m}{\partial r}\right| = \left|\frac{\partial\bar{u}}{\partial r}\right|[1 - Kg(r_*)],\quad r_* = \frac{r}{2R},\quad g(r_*) = 2\frac{\partial}{\partial r_*}\left(\frac{S_2-S_1}{S_2+S_1}\right), \tag{1.2}$$

where K = 0.5 is a coefficient connected with the above ratio of the Lagrangian and Eulerian scales. By changing this coefficient, we can henceforth correct the reduction in the modulus of the gradient of effective velocity $|\partial u_m/\partial r|$. If we make use of the approximate relation $(S_1 + S_2) \approx 2S_0$, then the function $g(r_*)$ takes the simpler form

$$g(r_*) = \frac{\partial}{\partial r_*}\left(\frac{S_2-S_1}{S_0}\right). \tag{1.3}$$

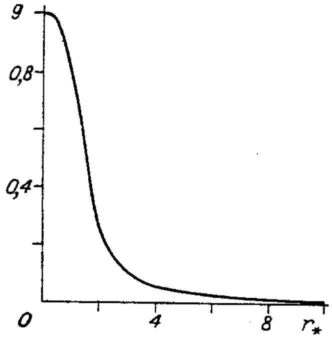

Fig. 3

Equation (1.3) can be expressed analytically. Solving the geometric problem of finding the areas S_1 and S_2 and then differentiating the relative difference between the areas with respect to r_*, after the transformations we find

$$g(r_*) = \frac{4}{9\,\mathrm{arctg}(\sqrt{8}) - \sqrt{8}}\left[(2r_* - 1)\,\mathrm{arctg}\left(\frac{2\sqrt{2r_*^2 - r_* - 1}}{2r_*^2 - r_* - 2}\right) + \right. \tag{1.4}$$
$$\left. + (2r_* + 1)\,\mathrm{arctg}\left(\frac{2\sqrt{2r_*^2 + r_* - 1}}{2r_*^2 + r_* - 2}\right) - \frac{2}{r_*}\left(\sqrt{2r_*^2 - r_* - 1} + \sqrt{2r_*^2 + r_* - 1}\right)\right].$$

Let us analyze the resulting solution at $r_* \to \infty$. Expanding (1.4) into a series in the small parameter r_*^{-1} and retaining the principal term of the expansion, after transformations we obtain the asymptotic relation

$$g(r_*) = \frac{16\sqrt{2}}{3r_*^2\,[9\,\mathrm{arctg}(\sqrt{8}) - \sqrt{8}]} \xrightarrow[r_* \to \infty]{} 0.$$

The proposed theory is not valid within the region $0 \le r_* < 1.5$, since in this case a different interpretation is needed for displacement of the mole in the direction of the symmetry axis. Let us try extrapolating the function $g(r_*)$ (1.4) to the axis ($r_* = 0$). Representing $g(r_*)$ in the form of a quadratic polynomial and requiring that the function and its derivative be continuous at $r_* = 1.5$ and be equal to zero at $r_* = 0$, after calculation we find that $g(0) \approx 1.0102$. Taking this result into account, we correct the extrapolated function as follows:

$$g(r_*) = 1 - [1 - g(1{,}5)](2r_*/3)^2, \quad 0 \leqslant r_* < 1{,}5. \tag{1.5}$$

Figure 3 shows the graph of the function $g(r_*)$ [see (1.4) and (1.5)]. It is evident that the function decreases monotonically with an increase in r_*.

Thus, the proposed interpretation of the Prandtl theory for axisymmetric flow makes it possible to introduce the effective modulus of the gradient of velocity ($|\partial u_m/\partial r|$), which is connected with the modulus of the gradient of mean velocity ($|\partial \bar{u}/\partial r|$) by (1.2).

2. Modification of Turbulence Models. First we will use the above result to improve the one-parameter turbulence model in [1]. In the case of an axisymmetric jet of an incompressible fluid (ρ = const) this model has the following form (with the averaging symbol henceforth being omitted)

$$u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial r} = \frac{2}{r}\frac{\partial}{\partial r}\left(r\varepsilon\frac{\partial \varepsilon}{\partial r}\right) + \varepsilon\left|\frac{\partial u}{\partial r}\right|\alpha. \tag{2.1}$$

It was assumed in [1] that $\alpha = 0.2$ for jet flows. In the present study, α is determined as

$$\alpha = 0{,}2\,[1 - Kg(0{,}5r/R)]. \tag{2.2}$$

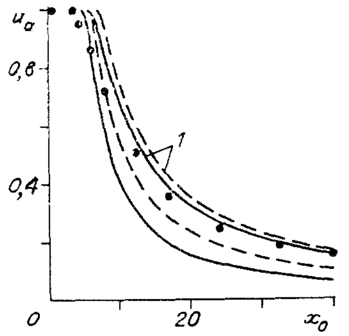

Fig. 4

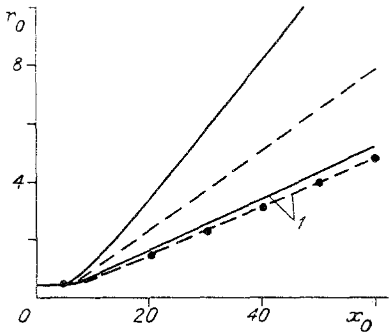

Fig. 5

By introducing the additional factor in the square brackets, we take into account the above-noted decrease in the modulus of the effective velocity gradient (see Eq. (1.2)).

We now establish the reliability between the radius of the mole R and the characteristics of the flow. We assume that the radius of the mole is equal to the integral scale of turbulence in the transverse direction:

$$R = L_r. \tag{2.3}$$

We then use the following experimental data, presented in [5], for the middle of the plane mixing layer of an incompressible fluid

$$\varepsilon/(Ux) = (1{,}6 - 2{,}2)\cdot 10^{-3},\ L_r \approx 0{,}04x,$$
$$\sqrt{q}/U \approx 0{,}19,\ \varepsilon|\partial u/\partial y| \approx 0{,}3q. \tag{2.4}$$

Here, U is the velocity of the jet; and q is turbulence energy. Using (2.3-2.4), after transformation we obtain the sought relationship between the radius of the mole and the characteristics of the flow:

$$R = \frac{\sqrt{\varepsilon}}{C\sqrt{|\partial u/\partial r|}},\quad C = 0{,}38\text{-}0{,}53. \tag{2.5}$$

Thus, a modified one-parameter turbulence model is described by relations (1.4-1.5), (2.1-2.2), (2.5) and two constants K and C.

We use a similar modification for the two-parameter model in [2], which includes equations for turbulence energy q and rate of dissipation ω. In the case of axisymmetric isobaric jet flow of an incompressible fluid (ρ = const), this model has the form

$$u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon\frac{\partial q}{\partial r}\right) + P - \omega; \tag{2.6}$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\varepsilon}{1{,}3}\frac{\partial \omega}{\partial r}\right) + \frac{\omega}{q}\left[1{,}45P - 2\omega\right]; \tag{2.7}$$

$$\varepsilon = 0{,}09q^2\omega^{-1},\ P = \varepsilon\left|\frac{\partial u}{\partial r}\right|^2. \tag{2.8}$$

The modification involves replacing the velocity gradient in the expression for the product P by its effective value P_m. Having used Eq. (1.2), we obtain an expression for the modified product:

$$P_m = \varepsilon\left|\frac{\partial u}{\partial r}\right|^2[1 - Kg(0{,}5r/R)]^2. \tag{2.9}$$

Thus, the modified two-parameter turbulence model is described by relations (1.4-1.5), (2.5-2.9) and the two constants C and K.

3. Calculation of an Axisymmetric Jet. To check the effectiveness of the modified turbulence models, we performed calculations of an isobaric submerged jet of an incompressible

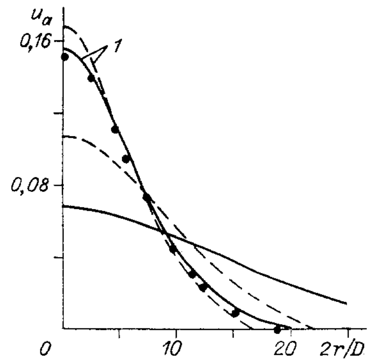

Fig. 6

fluid (ρ = const). The initial profiles of relative velocity $u_a \equiv u(x, r)/u(0, 0)$ and the relative coefficient of eddy viscosity $\varepsilon_a \equiv 2\varepsilon(x, r)/(u(0, 0)D)$ were assigned in the form

$$u_a(0, r) = \begin{cases} 1, & r/D \leqslant 0{,}45, \\ 10(1 - 2r/D), & 0{,}45 < r/D \leqslant 0{,}5, \\ 0, & r/D > 0{,}5, \end{cases}$$

$$\varepsilon_a(0, r) = \begin{cases} \varepsilon_0, & r/D \leqslant 0{,}5, \\ 0, & r/D > 0{,}5, \end{cases}$$

where D is the diameter of the nozzle; the value of ε_0 was found from the condition that the Reynolds number $Re_D \equiv VD/\varepsilon_0 = 10^4$. The velocity V was determined as follows:

$$V = \frac{2}{D}\sqrt{2\int_0^{D/2} u^2(0, t)\, t\, dt}.$$

The calculation was performed by means of a conservative finite-difference scheme [6] of first-order accuracy. The number of nodes of the mesh in the cross section of the jet was equal to 160 and 320. The difference between the results of calculations performed on coarse and fine meshes was no greater than 1%. The calculated results were compared with Rhode's experimental data [7]. When we used the modified one-parameter model, the best agreement between the theoretical and experimental data was obtained at C = 0.47 and K = 1, while the best agreement in the case of the two-parameter model was obtained at C = 0.62 and K = 1. The results of calculations performed with the one- and two parameter models are shown in Figs. 4-6 by the solid and dashed lines, respectively. Calculations were performed both with and without the modifications. The lines found with the modifications are designated by the number 1. Figures 4 and 5 show data on the change of relative velocity u_a and relative half-width $r_0 \equiv r_+/D$ along the axis ($x_0 \equiv x/D$). The relative half-width was determined from the velocity profile with the use of the relation $u(x, r_+) = 0.5u(x, 0)$.

Figure 6 shows the profile of relative velocity u_a in the section x/D = 40. It is evident that the modifications of the turbulence models significantly improve the accuracy with which mixing in an axisymmetric jet is described.

We also calculated the rate of expansion of the jet $r'_0 = dr_+/dx$ in its axial section (60 < x/D < 100). Whereas $r'_0 = 0.244$ before modification for the one-parameter model, $r'_0 = 0.091$ after the modification of this model. In the case of the two-parameter model, $r'_0 = 0.142$ before modification and $r'_0 = 0.085$ after modification.

It should be noted that for the two-parameter model in [8], the allowances made for axisymmetric flow were based on considerations different from those discussed in the present article.

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